Algebra I

The standards below outline the content for a one-year course in Algebra I. All students are expected to achieve the Algebra I standards. When planning for instruction, consideration will be given to the sequential development of concepts and skills by using concrete materials to assist students in making the transition from the arithmetic to the symbolic. Students should be helped to make connections and build relationships between algebra and arithmetic, geometry, and probability and statistics. Connections also should be made to other subject areas through practical applications. This approach to teaching algebra should help students attach meaning to the abstract concepts of algebra.

These standards require students to use algebra as a tool for representing and solving a variety of practical problems. Tables and graphs will be used to interpret algebraic expressions, equations, and inequalities and to analyze functions. Matrices will be used to organize and manipulate data.

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of functions; they provide a powerful tool for solving and verifying solutions to equations and inequalities.

Throughout the course, students should be encouraged to talk about mathematics, use the language and symbols of mathematics in representations and communication, discuss problems and problem solving, and develop their confidence in mathematics.

A.1 The student will solve multistep linear equations and inequalities in one variable, solve literal equations (formulas) for a given variable, and apply these skills to solve practical problems. Graphing calculators will be used to confirm algebraic solutions.

A.2 The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables. Students will choose an appropriate computational technique, such as mental mathematics, calculator, or paper and pencil.

A.3 The student will justify steps used in simplifying expressions and solving equations and inequalities. Justifications will include the use of concrete objects; pictorial representations; and the properties of real numbers, equality, and inequality.

A.4 The student will use matrices to organize and manipulate data, including matrix addition, subtraction, and scalar multiplication. Data will arise from business, industrial, and consumer situations.

A.5 The student will create and use tabular, symbolic, graphical, verbal, and physical representations to analyze a given set of data for the existence of a pattern, determine the domain and range of relations, and identify the relations that are functions.

A.6 The student will select, justify, and apply an appropriate technique to graph linear functions and linear inequalities in two variables. Techniques will include slope-intercept, x- and y-intercepts, graphing by transformation, and the use of the graphing calculator.

A.7 The student will determine the slope of a line when given an equation of the line, the graph of the line, or two points on the line. Slope will be described as rate of change and will be positive, negative, zero, or undefined. The graphing calculator will be used to investigate the effect of changes in the slope on the graph of the line.

A.8 The student will write an equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line.

A.9 The student will solve systems of two linear equations in two variables both algebraically and graphically and apply these techniques to solve practical problems. Graphing calculators will be used both as a primary tool for solution and to confirm an algebraic solution.

A.10 The student will apply the laws of exponents to perform operations on expressions with integral exponents, using scientific notation when appropriate.

A.11 The student will add, subtract, and multiply polynomials and divide polynomials with monomial divisors, using concrete objects, pictorial and area representations, and algebraic manipulations.

A.12 The student will factor completely first- and second-degree binomials and trinomials in one or two variables. The graphing calculator will be used as a tool for factoring and for confirming algebraic factorizations.

A.13 The student will express the square root of a whole number in simplest radical form and approximate square roots to the nearest tenth.

A.14 The student will solve quadratic equations in one variable both algebraically and graphically. Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

A.15 The student will, given a rule, find the values of a function for elements in its domain and locate the zeros of the function both algebraically and with a graphing calculator. The value of $f(x)$ will be related to the ordinate on the graph.

A.16 The student will, given a set of data points, write an equation for a line of best fit and use the equation to make predictions.

A.17 The student will compare and contrast multiple one-variable data sets, using statistical techniques that include measures of central tendency, range, and box-and-whisker graphs.

A.18 The student will analyze a relation to determine whether a direct variation exists and represent it algebraically and graphically, if possible.

Geometry

This course is designed for students who have successfully completed the standards for Algebra I. The course includes, among other things, properties of geometric figures, trigonometric relationships, and reasoning to justify conclusions. Methods of justification will include paragraph proofs, two-column proofs, indirect proofs, coordinate proofs, and verbal arguments. A gradual development of formal proof is encouraged. Inductive and intuitive approaches to proof as well as deductive axiomatic methods should be used.

This set of standards includes emphasis on two- and three-dimensional reasoning skills, coordinate and transformational geometry, and the use of geometric models to solve problems. A variety of applications and some general problem-solving techniques including algebraic skills, should be used to implement these standards. Calculators, computers, graphing utilities (graphing calculators or computer graphing simulators), dynamic geometry software, and other appropriate technology tools will be used to assist in teaching and learning. Any technology that will enhance student learning should be used.

G.1 The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
- a) identifying the converse, inverse, and contrapositive of a conditional statement;
- b) translating a short verbal argument into symbolic form;
- c) using Venn diagrams to represent set relationships; and
- d) using deductive reasoning, including the law of syllogism.

G.2 The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation. This will include
- a) investigating and using formulas for finding distance, midpoint, and slope;
- b) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
- c) determining whether a figure has been translated, reflected, or rotated.

G.3 The student will solve practical problems involving complementary, supplementary, and congruent angles that include vertical angles, angles formed when parallel lines are cut by a transversal, and angles in polygons.

G.4 The student will use the relationships between angles formed by two lines cut by a transversal to determine if two lines are parallel and verify, using algebraic and coordinate methods as well as deductive proofs.

G.5 The student will
- a) investigate and identify congruence and similarity relationships between triangles; and
- b) prove two triangles are congruent or similar, given information in the form of a figure or statement, using algebraic and coordinate as well as deductive proofs.

G.6 The student, given information concerning the lengths of sides and/or measures of angles, will apply the triangle inequality properties to determine whether a triangle exists and to order sides and angles. These concepts will be considered in the context of practical situations.

G.7 The student will solve practical problems involving right triangles by using the Pythagorean Theorem, properties of special right triangles, and right triangle trigonometry. Solutions will be expressed in radical form or as decimal approximations.

G.8 The student will
a) investigate and identify properties of quadrilaterals involving opposite sides and angles, consecutive sides and angles, and diagonals;
b) prove these properties of quadrilaterals, using algebraic and coordinate methods as well as deductive reasoning; and
c) use properties of quadrilaterals to solve practical problems.

G.9 The student will use measures of interior and exterior angles of polygons to solve problems. Tessellations and tiling problems will be used to make connections to art, construction, and nature.

G.10 The student will investigate and solve practical problems involving circles, using properties of angles, arcs, chords, tangents, and secants. Problems will include finding arc length and the area of a sector, and may be drawn from applications of architecture, art, and construction.

G.11 The student will construct a line segment congruent to a given line segment, the bisector of a line segment, a perpendicular to a given line from a point not on the line, a perpendicular to a given line at a point on the line, the bisector of a given angle, and an angle congruent to a given angle.

G.12 The student will make a model of a three-dimensional figure from a two-dimensional drawing and make a two-dimensional representation of a three-dimensional object. Models and representations will include scale drawings, perspective drawings, blueprints, or computer simulations.

G.13 The student will use formulas for surface area and volume of three-dimensional objects to solve practical problems. Calculators will be used to find decimal approximations for results.

G.14 The student will
a) use proportional reasoning to solve practical problems, given similar geometric objects; and
b) determine how changes in one dimension of an object affect area and/or volume of the object.

Algebra II

The standards below outline the content for a one-year course in Algebra II. Students enrolled in Algebra II are assumed to have mastered those concepts outlined in the Algebra I standards. A thorough treatment of advanced algebraic concepts is provided through the study of functions, "families of functions," equations, inequalities, systems of equations and inequalities, polynomials, rational expressions, complex numbers, matrices, and sequences and series. Emphasis will be placed on practical applications and modeling throughout the course of study. Oral and written communication concerning the language of algebra, logic of procedures, and interpretation of results also should permeate the course.

These standards include a transformational approach to graphing functions. Transformational graphing uses translation, reflection, dilation, and rotation to generate a "family of graphs" from a given graph and builds a strong connection between algebraic and graphic representations of functions. Students will vary the coefficients and constants of an equation, observe the changes in the graph of the equation, and make generalizations that can be applied to many graphs.

Graphing utilities (graphing calculators or computer graphing simulators), computers, spreadsheets, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through mathematical modeling and aid in the investigation and study of functions. They also provide an effective tool for solving/verifying equations and inequalities. Any other available technology that will enhance student learning should be used.

AII.1 The student will identify field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets, complex numbers, and matrices.

AII.2 The student will add, subtract, multiply, divide, and simplify rational expressions, including complex fractions.

AII.3 The student will
a) add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables and expressions containing rational exponents; and
b) write radical expressions as expressions containing rational exponents and vice versa.

AII.4 The student will solve absolute value equations and inequalities graphically and algebraically. Graphing calculators will be used as a primary method of solution and to verify algebraic solutions.

AII.5 The student will identify and factor completely polynomials representing the difference of squares, perfect square trinomials, the sum and difference of cubes, and general trinomials.

AII.6 The student will select, justify, and apply a technique to solve a quadratic equation over the set of complex numbers. Graphing calculators will be used for solving and for confirming the algebraic solutions.

AII.7 The student will solve equations containing rational expressions and equations containing radical expressions algebraically and graphically. Graphing calculators will be used for solving and for confirming the algebraic solutions.

AII.8 The student will recognize multiple representations of functions (linear, quadratic, absolute value, step, and exponential functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.

AII.9 The student will find the domain, range, zeros, and inverse of a function; the value of a function for a given element in its domain; and the composition of multiple functions. Functions will include exponential, logarithmic, and those that have domains and ranges that are limited and/or discontinuous. The graphing calculator will be used as a tool to assist in investigation of functions.

AII.10 The student will investigate and describe through the use of graphs the relationships between the solution of an equation, zero of a function, x-intercept of a graph, and factors of a polynomial expression.

AII.11 The student will use matrix multiplication to solve practical problems. Graphing calculators or computer programs with matrix capabilities will be used to find the product.

AII.12 The student will represent problem situations with a system of linear equations and solve the system, using the inverse matrix method. Graphing calculators or computer programs with matrix capability will be used to perform computations.

AII.13 The student will solve practical problems, using systems of linear inequalities and linear programming, and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.

AII.14 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.

AII.15 The student will recognize the general shape of polynomial, exponential, and logarithmic functions. The graphing calculator will be used as a tool to investigate the shape and behavior of these functions.

AII.16 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems, including writing the first n terms, finding the n^{th} term, and evaluating summation formulas. Notation will include Σ and a_n.

AII.17 The student will perform operations on complex numbers and express the results in simplest form. Simplifying results will involve using patterns of the powers of *i*.

AII.18 The student will identify conic sections (circle, ellipse, parabola, and hyperbola) from his/her equations. Given the equations in *(h, k)* form, the student will sketch graphs of conic sections, using transformations.

AII.19 The student will collect and analyze data to make predictions and solve practical problems. Graphing calculators will be used to investigate scatterplots and to determine the equation for a curve of best fit. Models will include linear, quadratic, exponential, and logarithmic functions.

AII.20 The student will identify, create, and solve practical problems involving inverse variation and a combination of direct and inverse variations.

Trigonometry

The standards below outline the content for a one-semester course in trigonometry. Students enrolled in trigonometry are assumed to have mastered those concepts outlined in the Algebra II standards. A thorough treatment of trigonometry is provided through the study of trigonometric definitions, applications, graphing, and solving trigonometric equations and inequalities. Emphasis should also be placed on using connections between right triangle ratios, trigonometric functions, and circular functions. In addition, applications and modeling should be included throughout the course of study. Emphasis should also be placed on oral and written communication concerning the language of mathematics, logic of procedure, and interpretation of results.

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of trigonometric functions and their inverses. They also provide a powerful tool for solving/verifying trigonometric equations and inequalities.

T.1 The student will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of an angle in standard position, given a point, other than the origin, on the terminal side of the angle. Circular function definitions will be connected with trigonometric function definitions.

T.2 The student, given the value of one trigonometric function, will find the values of the other trigonometric functions. Properties of the unit circle and definitions of circular functions will be applied.

T.3 The student will find without the aid of a calculating utility the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting radians to degrees and vice versa.

T.4 The student will find with the aid of a calculator the value of any trigonometric function and inverse trigonometric function.

T.5 The student will verify basic trigonometric identities and make substitutions, using the basic identities.

T.6 The student, given one of the six trigonometric functions in standard form [e.g., $y = A \sin (Bx + C) + D$, where $A, B, C,$ and D are real numbers], will

a) state the domain and the range of the function;
b) determine the amplitude, period, phase shift, and vertical shift; and
c) sketch the graph of the function by using transformations for at least a one-period interval.

The graphing calculator will be used to investigate the effect of changing $A, B, C,$ and D on the graph of a trigonometric function.

T.7 The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.

T.8 The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities. Graphing utilities will be used to solve equations, check for reasonableness of results, and verify algebraic solutions.

T.9 The student will identify, create, and solve practical problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

Algebra II and Trigonometry

The standards for this combined course in Algebra II and Trigonometry include all of the standards listed for Algebra II and Trigonometry. This course is designed for advanced students who are capable of a more rigorous course at an accelerated pace. The standards listed for this course provide the foundation for students to pursue a sequence of advanced mathematical studies from Mathematical Analysis to Advanced Placement Calculus.

AII/T.1 The student will identify field properties, axioms of equality and inequality, and properties of order that are valid for the set of real numbers and its subsets, complex numbers, and matrices.

AII/T.2 The student will add, subtract, multiply, divide, and simplify rational expressions, including complex fractions.

AII/T.3 The student will
a) add, subtract, multiply, divide, and simplify radical expressions containing positive rational numbers and variables and expressions containing rational exponents; and
b) write radical expressions as expressions containing rational exponents and vice versa.

AII/T.4 The student will solve absolute value equations and inequalities graphically and algebraically. Graphing calculators will be used as a primary method of solution and to verify algebraic solutions.

AII/T.5 The student will identify and factor completely polynomials representing the difference of squares, perfect square trinomials, the sum and difference of cubes, and general trinomials.

AII/T.6 The student will select, justify, and apply a technique to solve a quadratic equation over the set of complex numbers. Graphing calculators will be used for solving and for confirming the algebraic solutions.

AII/T.7 The student will solve equations containing rational expressions and equations containing radical expressions algebraically and graphically. Graphing calculators will be used for solving and for confirming the algebraic solutions.

AII/T.8 The student will recognize multiple representations of functions (linear, quadratic, absolute value, step, and exponential functions) and convert between a graph, a table, and symbolic form. A transformational approach to graphing will be employed through the use of graphing calculators.

AII/T.9 The student will find the domain, range, zeros, and inverse of a function; the value of a function for a given element in its domain; and the composition of multiple functions. Functions will include exponential, logarithmic, and those that have domains and ranges that are limited and/or discontinuous. The graphing calculator will be used as a tool to assist in investigation of functions.

AII/T.10 The student will investigate and describe through the use of graphs the relationships between the solution of an equation, zero of a function, x-intercept of a graph, and factors of a polynomial expression.

AII/T.11 The student will use matrix multiplication to solve practical problems. Graphing calculators or computer programs with matrix capabilities will be used to find the product.

AII/T.12 The student will represent problem situations with a system of linear equations and solve the system, using the inverse matrix method. Graphing calculators or computer programs with matrix capability will be used to perform computations.

AII/T.13 The student will solve practical problems, using systems of linear inequalities and linear programming, and describe the results both orally and in writing. A graphing calculator will be used to facilitate solutions to linear programming problems.

AII/T.14 The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. The graphing calculator will be used as a tool to visualize graphs and predict the number of solutions.

AII/T.15 The student will recognize the general shape of polynomial, exponential, and logarithmic functions. The graphing calculator will be used as a tool to investigate the shape and behavior of these functions.

AII/T.16 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve practical problems, including writing the first n terms, finding the n^{th} term, and evaluating summation formulas. Notation will include Σ and a_n.

AII/T.17 The student will perform operations on complex numbers and express the results in simplest form. Simplifying results will involve using patterns of the powers of i.

AII/T.18 The student will identify conic sections (circle, ellipse, parabola, and hyperbola) from his/her equations. Given the equations in *(h, k)* form, the student will sketch graphs of conic sections, using transformations.

AII/T.19 The student will collect and analyze data to make predictions and solve practical problems. Graphing calculators will be used to investigate scatterplots and to determine the equation for a curve of best fit. Models will include linear, quadratic, exponential, and logarithmic functions.

AII/T.20 The student will identify, create, and solve practical problems involving inverse variation and a combination of direct and inverse variations.

AII/T.21 The student will use the definitions of the six trigonometric functions to find the sine, cosine, tangent, cotangent, secant, and cosecant of an angle in standard position, given a point, other than the origin, on the terminal side of the angle. Circular function definitions will be connected with trigonometric function definitions.

AII/T.22 The student, given the value of one trigonometric function, will find the values of the other trigonometric functions. Properties of the unit circle and definitions of circular functions will be applied.

AII/T.23 The student will find without the aid of a calculating utility the values of the trigonometric functions of the special angles and their related angles as found in the unit circle. This will include converting radians to degrees and vice versa.

AII/T.24 The student will find with the aid of a calculator the value of any trigonometric function and inverse trigonometric function.

AII/T.25 The student will verify basic trigonometric identities and make substitutions, using the basic identities.

AII/T.26 The student, given one of the six trigonometric functions in standard form [e.g., $y = A \sin (Bx + C) + D$, where $A, B, C,$ and D are real numbers], will

a) state the domain and the range of the function;
b) determine the amplitude, period, phase shift, and vertical shift; and
c) sketch the graph of the function by using transformations for at least a one-period interval.

The graphing calculator will be used to investigate the effect of changing $A, B, C,$ and D on the graph of a trigonometric function.

AII/T.27 The student will identify the domain and range of the inverse trigonometric functions and recognize the graphs of these functions. Restrictions on the domains of the inverse trigonometric functions will be included.

AII/T.28 The student will solve trigonometric equations that include both infinite solutions and restricted domain solutions and solve basic trigonometric inequalities. Graphing utilities will be used to solve equations, check for reasonableness of results, and verify algebraic solutions.

AII/T.29 The student will identify, create, and solve practical problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

Computer Mathematics

This course is intended to provide students with experiences in using computer programming techniques and skills to solve problems that can be set up as mathematical models. Students enrolled in Computer Mathematics are assumed to have studied the concepts and skills in Algebra I and beginning geometry. Students who successfully complete the standards for this course may earn credit toward meeting the mathematics graduation requirement. It is recognized that many students will gain computer skills in other mathematics courses or in a separate curriculum outside of mathematics and prior to high school. In such cases, the standards indicated by an asterisk (*) should be included in the student's course of study and treated as a review.

Even though computer ideas should be introduced in the context of mathematical concepts, problem solving per se should be developed in the most general sense, making the techniques applicable by students in many other environments. Strategies include defining the problem; developing, refining, and implementing a plan; and testing and revising the solution. Programming, ranging from simple programs involving only a few lines to complex programs involving subprograms, should permeate the entire course and may include programming a graphing calculator or scripting a problem solution in a database or spreadsheet. Programming concepts, problem-solving strategies, and mathematical applications should be integrated throughout the course.

These standards identify fundamental principles and concepts in the field of computer science that will be used within the context of mathematical problem solving in a variety of applications. As students develop and refine skills in logic, organization, and precise expression, they will apply those skills to enhance learning in all disciplines.

COM.1 The student will apply programming techniques and skills to solve practical problems in mathematics arising from consumer, business, other applications in mathematics. Problems will include opportunities for students to analyze data in charts, graphs, and tables and to use their knowledge of equations, formulas, and functions to solve these problems.

*COM.2 The student will design, write, test, debug, and document a program. Programming documentation will include pre-conditions and post-conditions of program segments, input/output specifications, the step-by-step plan, the test data, a sample run, and the program listing with appropriately placed comments.

*COM.3 The student will write program specifications that define the constraints of a given problem. These specifications will include descriptions of pre-conditions, post-conditions, the desired output, analysis of the available input, and an indication as to whether or not the problem is solvable under the given conditions.

*COM.4 The student will design a step-by-step plan (algorithm) to solve a given problem. The plan will be in the form of a program flowchart, pseudo code, hierarchy chart, and/or data-flow diagram.

*COM.5 The student will divide a given problem into manageable sections (modules) by task and implement the solution. The modules will include an appropriate user-defined function, subroutines, and procedures. Enrichment topics might include user-defined libraries (units) and object-oriented programming.

*COM.6 The student will design and implement the input phase of a program, which will include designing screen layout and getting information into the program by way of user interaction, data statements, and/or file input. The input phase also will include methods of filtering out invalid data (error trapping).

*COM.7 The student will design and implement the output phase of a computer program, which will include designing output layout, accessing a variety of output devices, using output statements, and labeling results.

COM.8 The student will design and implement computer graphics, which will include topics appropriate for the available programming environment as well as student background. Students will use graphics as an end in itself, as an enhancement to other output, and as a vehicle for reinforcing programming techniques.

COM.9 The student will define simple variable data types that include integer, real (fixed and scientific notation), character, string, and Boolean.

COM.10 The student will use appropriate variable data types, including integer, real (fixed and scientific notation), character, string, and Boolean. This will also include variables representing structured data types.

*COM.11 The student will describe the way the computer stores, accesses, and processes variables, including the following topics: the use of variables versus constants, variables addresses, pointers, parameter passing, scope of variables, and local versus global variables.

COM.12 The student will translate a mathematical expression into a computer statement, which involves writing assignment statements and using the order of operations.

COM.13 The student will select and implement built-in (library) functions in processing data.

COM.14 The student will implement conditional statements that include “if/then” statements, “if/then/else” statements, case statements, and Boolean logic.

COM.15 The student will implement loops, including iterative loops. Other topics will include single entry point, single exit point, pre-conditions, and post-conditions.

*COM.16 The student will select and implement appropriate data structures, including arrays (one-dimensional and/or multidimensional), files, and records. Implementation will include creating the data structure, putting information into the structure, and retrieving information from the structure.

*COM.17 The student will implement pre-existing algorithms, including sort routines, search routines, and simple animation routines.

COM.18 The student will test a program, using an appropriate set of data. The set of test data should be appropriate and complete for the type of program being tested.

COM.19 The student will debug a program, using appropriate techniques (e.g., appropriately placed controlled breaks, the printing of intermediate results, and other debugging tools available in the programming environment), and identify the difference between syntax errors and logic errors.

COM.20 The student will design, write, test, debug, and document a complete structured program that requires the synthesis of many of the concepts contained in previous standards.

Probability and Statistics

The following standards outline the content of a one-year course in Probability and Statistics. If a one-semester course is desired, the standards with an asterisk (*) would apply. Students enrolled in this course are assumed to have mastered the concepts identified in the Standards of Learning for Algebra II. The purpose of the course is to present basic concepts and techniques for collecting and analyzing data, drawing conclusions, and making predictions.

A graphing calculator is essential for every student taking the Probability and Statistics course and is required for the Advanced Placement Statistics Examination. The calculator may not fully substitute for a computer, however. In the absence of a computer for student use, teachers may provide students with examples of computer output generated by a statistical software package.

*PS.1 The student will analyze graphical displays of data, including dotplots, stemplots, and histograms, to identify and describe patterns and departures from patterns, using central tendency, spread, clusters, gaps, and outliers. Appropriate technology will be used to create graphical displays.

*PS.2 The student will analyze numerical characteristics of univariate data sets to describe patterns and departure from patterns, using mean, median, mode, variance, standard deviation, interquartile range, range, and outliers. Appropriate technology will be used to calculate statistics.

*PS.3 The student will compare distributions of two or more univariate data sets, analyzing center and spread (within group and between group variations), clusters and gaps, shapes, outliers, or other unusual features. Appropriate technology will be used to generate graphical displays.

*PS.4 The student will analyze scatterplots to identify and describe the relationship between two variables, using shape; strength of relationship; clusters; positive, negative, or no association; outliers; and influential points. Appropriate technology will be used to generate scatterplots and identify outliers and influential points.

PS.5 The student will find and interpret linear correlation, use the method of least squares regression to model the linear relationship between two variables, and use the residual plots to assess linearity. Appropriate technology will be used to compute correlation coefficients and residual plots.

PS.6 The student will make logarithmic and power transformations to achieve linearity. Appropriate technology will be used.

PS.7 The student, using two-way tables, will analyze categorical data to describe patterns and departure from patterns and to find marginal frequency and relative frequencies, including conditional frequencies.

*PS.8 The student will describe the methods of data collection in a census, sample survey, experiment, and observational study and identify an appropriate method of solution for a given problem setting.

*PS.9 The student will plan and conduct a survey. The plan will address sampling techniques (e.g., simple random and stratified) and methods to reduce bias.

PS.10 The student will plan and conduct an experiment. The plan will address control, randomization, and measurement of experimental error.

*PS.11 The student will compute and distinguish between permutations and combinations and use technology for applications.

*PS.12 The student will identify and describe two or more events as complementary, dependent, independent, and/or mutually exclusive.

*PS.13 The student will find probabilities (relative frequency and theoretical), including conditional probabilities for events that are either dependent or independent, by applying the "law of large numbers" concept, the addition rule, and the multiplication rule.

*PS.14 The student will develop, interpret, and apply the binomial probability distribution for discrete random variables, including computing the mean and standard deviation for the binomial variable.

PS.15 The student will simulate probability distributions, including binomial and geometric.

PS.16 The student will identify random variables as independent or dependent and find the mean and standard deviations for sums and differences of independent random variables.

*PS.17 The student will identify properties of a normal distribution and apply the normal distribution to determine probabilities, using a table or graphing calculator.

*PS.18 The student, given data from a large sample, will find and interpret point estimates and confidence intervals for parameters. The parameters will include proportion and mean, difference between two proportions, and difference between two means (independent and paired).

PS.19 The student will apply and interpret the logic of a hypothesis-testing procedure. Tests will include large sample test for proportion, mean, difference between two proportions, and difference between two means (independent and paired) and Chi-squared test for goodness of fit, homogeneity of proportions, and independence.

PS.20 The student will identify the meaning of sampling distribution with reference to random variable, sampling statistic, and parameter and explain the Central Limit Theorem. This will include sampling distribution of a sample proportion, a sample mean, a difference between two sample proportions, and a difference between two sample means.

PS.21 The student will identify properties of a t-distribution and apply t-distributions to single-sample and two-sample (independent and matched pairs) t-procedures, using tables or graphing calculators.

Discrete Mathematics

The following standards outline the content of a one-year course in Discrete Mathematics. If a one-semester course is desired, the standards with an asterisk (*) would apply. Students enrolled in Discrete Mathematics are assumed to have mastered the concepts outlined in the Standards of Learning for Algebra II.

Discrete mathematics may be described as the study of mathematical properties of sets and systems that have a countable (discrete) number of elements. With the advent of modern technology, discrete (discontinuous) models have become as important as continuous models. In this course, the main focus is problem solving in a discrete setting. Techniques that are not considered in the current traditional courses of algebra, geometry, and calculus will be utilized. As students solve problems, they will analyze and determine whether or not a solution exists (existence problems), investigate how many solutions exist (counting problems), and focus on finding the best solution (optimization problems). Connections will be made to other disciplines. The importance of discrete mathematics has been influenced by computers. Modern technology (graphing calculators and/or computers) will be an integral component of this course.

*DM.1 The student will model problems, using vertex-edge graphs. The concepts of valence, connectedness, paths, planarity, and directed graphs will be investigated. Adjacency matrices and matrix operations will be used to solve problems (e.g., food chains, number of paths).

*DM.2 The student will solve problems through investigation and application of circuits, cycles, Euler Paths, Euler Circuits, Hamilton Paths, and Hamilton Circuits. Optimal solutions will be sought using existing algorithms and student-created algorithms.

*DM.3 The student will apply graphs to conflict-resolution problems, such as map coloring, scheduling, matching, and optimization. Graph coloring and chromatic number will be used.

*DM.4 The student will apply algorithms, such as Kruskal's, Prim's, or Dijkstra's, relating to trees, networks, and paths. Appropriate technology will be used to determine the number of possible solutions and generate solutions when a feasible number exists.

*DM.5 The student will use algorithms to schedule tasks in order to determine a minimum project time. The algorithms will include critical path analysis, the list-processing algorithm, and student-created algorithms.

*DM.6 The student will solve linear programming problems. Appropriate technology will be used to facilitate the use of matrices, graphing techniques, and the Simplex method of determining solutions.

*DM.7 The student will analyze and describe the issue of fair division (e.g., cake cutting, estate division). Algorithms for continuous and discrete cases will be applied.

DM.8 The student will investigate and describe weighted voting and the results of various election methods. These may include approval and preference voting as well as plurality, majority, run-off, sequential run-off, Borda count, and Condorcet winners.

DM.9 The student will identify apportionment inconsistencies that apply to issues such as salary caps in sports and allocation of representatives to Congress. Historical and current methods will be compared.

DM.10 The student will use the recursive process and difference equations with the aid of appropriate technology to generate
a) compound interest;
b) sequences and series;
c) fractals;
d) population growth models; and
e) the Fibonacci sequence.

DM.11 The student will describe and apply sorting algorithms and coding algorithms used in storing, processing, and communicating information. These will include
a) bubble sort, merge sort, and network sort; and
b) ISBN, UPC, Zip, and banking codes.

DM.12 The student will select, justify, and apply an appropriate technique to solve a logic problem. Techniques will include Venn diagrams, truth tables, and matrices.

DM.13 The student will apply the formulas of combinatorics in the areas of
a) the Fundamental (Basic) Counting Principle;
b) knapsack and bin-packing problems;
c) permutations and combinations; and
d) the pigeonhole principle.

Mathematical Analysis

The standards below outline the content for a one-year course in Mathematical Analysis. Students enrolled in Mathematical Analysis are assumed to have mastered Algebra II concepts and have some exposure to trigonometry. Mathematical Analysis develops students' understanding of algebraic and transcendental functions, parametric and polar equations, sequences and series, and vectors. The content of this course serves as appropriate preparation for a calculus course.

Graphing calculators, computers, and other appropriate technology tools will be used to assist in teaching and learning. Graphing utilities enhance the understanding of realistic applications through modeling and aid in the investigation of functions and their inverses. They also provide a powerful tool for solving and verifying equations and inequalities.

MA.1 The student will investigate and identify the characteristics of polynomial and rational functions and use these to sketch the graphs of the functions. This will include determining zeros, upper and lower bounds, *y*-intercepts, symmetry, asymptotes, intervals for which the function is increasing or decreasing, and maximum or minimum points. Graphing utilities will be used to investigate and verify these characteristics.

MA.2 The student will find compositions of functions and inverses of functions. Analytical methods and graphing utilities will be used to investigate and verify the domain and range of resulting functions.

MA.3 The student will investigate and describe the continuity of functions, using graphs. The functions will include absolute value, piecewise, and step functions.

MA.4 The student will expand binomials having positive integral exponents through the use of the Binomial Theorem, the formula for combinations, and Pascal's Triangle.

MA.5 The student will solve practical problems involving arithmetic and geometric sequences and series. This will include finding the sum (sigma notation included) of finite and infinite convergent series that will lead to an intuitive approach to a limit.

MA.6 The student will use mathematical induction to prove formulas/statements.

MA.7 The student will find the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. A graphing utility will be used to verify intuitive reasoning, algebraic methods, and numerical substitution.

MA.8 The student will investigate and identify the characteristics of conic section equations in *(h, k)* and standard forms. The techniques of translation and rotation of axes in the coordinate plane will be used to graph conic sections.

MA.9 The student will investigate and identify the characteristics of exponential and logarithmic functions in order to graph these functions and solve equations and practical problems. This will include the role of e, natural and common logarithms, laws of exponents and logarithms, and the solution of logarithmic and exponential equations. Graphing utilities will be used to investigate and verify the graphs and solutions.

MA.10 The student will investigate and identify the characteristics of the graphs of polar equations, using graphing utilities. This will include classification of polar equations, the effects of changes in the parameters in polar equations, conversion of complex numbers from rectangular form to polar form and vice versa, and the intersection of the graphs of polar equations.

MA.11 The student will perform operations with vectors in the coordinate plane and solve practical problems using vectors. This will include the following topics: operations of addition, subtraction, scalar multiplication, and inner (dot) product; norm of a vector; unit vector; graphing; properties; simple proofs; complex numbers (as vectors); and perpendicular components.

MA.12 The student will use parametric equations to model and solve application problems. Graphing utilities will be used to develop an understanding of the graph of parametric equations.

MA.13 The student will identify, create, and solve practical problems involving triangles. Techniques will include using the trigonometric functions, the Pythagorean Theorem, the Law of Sines, and the Law of Cosines.

Advanced Placement Calculus

This course is intended for students who have a thorough knowledge of analytic geometry and elementary functions in addition to college preparatory algebra, geometry, and trigonometry. The purpose of the course is to prepare students for advanced placement in college calculus. These standards incorporate The College Board Advanced Placement Course Description Syllabus. Teachers should update course content as changes occur in future College Board publications.

As mandated by The College Board, graphing calculators are required for this course. Computers should be used when feasible by students and by teachers. Any other available technology tools that will enhance student learning should be used. Instructional activities that engage students in solving application problems of varying complexities are encouraged.

APC.1 The student will define and apply the properties of elementary functions, including algebraic, trigonometric, exponential, and composite functions and their inverses, and graph these functions, using a graphing calculator. Properties of functions will include domains, ranges, combinations, odd, even, periodicity, symmetry, asymptotes, zeros, upper and lower bounds, and intervals where the function is increasing or decreasing.

APC.2 The student will define and apply the properties of limits of functions. Limits will be evaluated graphically and algebraically. This will include

a) limits of a constant;
b) limits of a sum, product, and quotient;
c) one-sided limits; and
d) limits at infinity, infinite limits, and non-existent limits. *

*AP Calculus BC will include l'Hopital's Rule, which will be used to find the limit of functions whose limits yield the indeterminate forms: 0/0 and ∞/∞.

APC.3 The student will use limits to define continuity and determine where a function is continuous or discontinuous. This will include

a) continuity in terms of limits;
b) continuity at a point and over a closed interval;
c) application of the Intermediate Value Theorem and the Extreme Value Theorem; and
d) geometric understanding and interpretation of continuity and discontinuity.

APC.4 The student will investigate asymptotic and unbounded behavior in functions. This will include

a) describing and understanding asymptotes in terms of graphical behavior and limits involving infinity; and
b) comparing relative magnitudes of functions and their rates of change.

APC.5 The student will investigate derivatives presented in graphic, numerical, and analytic contexts and the relationship between continuity and differentiability. The derivative will be defined as the limit of the difference quotient and interpreted as an instantaneous rate of change.

APC.6 The student will investigate the derivative at a point on a curve. This will include
a) finding the slope of a curve at a point, including points at which the tangent is vertical and points at which there are no tangents;
b) using local linear approximation to find the slope of a tangent line to a curve at the point;
c) defining instantaneous rate of change as the limit of average rate of change; and
d) approximating rate of change from graphs and tables of values.

APC.7 The student will analyze the derivative of a function as a function in itself. This will include
a) comparing corresponding characteristics of the graphs of f, f', and f'';
b) defining the relationship between the increasing and decreasing behavior of f and the sign of f';
c) translating verbal descriptions into equations involving derivatives and vice versa;
d) analyzing the geometric consequences of the Mean Value Theorem;
e) defining the relationship between the concavity of f and the sign of f''; and
f) identifying points of inflection as places where concavity changes and finding points of inflection.

APC.8 The student will apply the derivative to solve problems. This will include
a) analysis of curves and the ideas of concavity and monotonicity;
b) optimization involving global and local extrema;
c) modeling of rates of change and related rates;
d) use of implicit differentiation to find the derivative of an inverse function;
e) interpretation of the derivative as a rate of change in applied contexts, including velocity, speed, and acceleration; and
f) differentiation of nonlogarithmic functions, using the technique of logarithmic differentiation. *

* AP Calculus BC will also apply the derivative to solve problems. This will include
a) analysis of planar curves given in parametric form, polar form, and vector form, including velocity and acceleration vectors;
b) numerical solution of differential equations, using Euler's method;
c) l'Hopital's Rule to test the convergence of improper integrals and series; and
d) geometric interpretation of differential equations via slope fields and the relationship between slope fields and the solution curves for the differential equations.

APC.9 The student will apply formulas to find derivatives. This will include
a) derivatives of algebraic, trigonometric, exponential, logarithmic, and inverse trigonometric functions;
b) derivations of sums, products, quotients, inverses, and composites (chain rule) of elementary functions;
c) derivatives of implicitly defined functions; and
d) higher order derivatives of algebraic, trigonometric, exponential, and logarithmic, functions. *

* AP Calculus BC will also include finding derivatives of parametric, polar, and vector functions.

APC.10 The student will use Riemann sums and the Trapezoidal Rule to approximate definite integrals of functions represented algebraically, graphically, and by a table of values and will interpret the definite integral as the accumulated rate of change of a quantity over an interval interpreted as the change of the quantity over the interval

$$\int_a^b f'(x)\,dx = f(b) - f(a).$$

Riemann sums will use left, right, and midpoint evaluation points over equal subdivisions.

APC.11 The student will find antiderivatives directly from derivatives of basic functions and by substitution of variables (including change of limits for definite integrals). *

* AP Calculus BC will also include finding antiderivatives by parts and simple partial fractions (nonrepeating linear factors only), and finding improper integrals as limits of definite integrals.

* AP Calculus BC will also solve logistic differential equations and use them in modeling.

APC.12 The student will identify the properties of the definite integral. This will include additivity and linearity, the definite integral as an area, and the definite integral as a limit of a sum as well as the fundamental theorem:

$$d/dx \int_a^x f(t)\,d(t) = f(x).$$

APC.13 The student will use the Fundamental Theorem of Calculus to evaluate definite integrals, represent a particular antiderivative, and facilitate the analytical and graphical analysis of functions so defined.

APC.14 The student will find specific antiderivatives, using initial conditions (including applications to motion along a line). Separable differential equations will be solved and used in modeling (in particular, the equation $y' = ky$ and exponential growth).

APC.15 The student will use integration techniques and appropriate integrals to model physical, biological, and economic situations. The emphasis will be on using the integral of a rate of change to give accumulated change or on using the method of setting up an approximating Riemann sum and representing its limit as a definite integral. Specific applications will include

a) the area of a region;
b) the volume of a solid with known cross-section;
c) the average value of a function; and
d) the distance traveled by a particle along a line. *

* AP Calculus BC will include finding the area of a region (including a region bounded by polar curves) and finding the length of a curve (including a curve given in parametric form).

APC.16 The student will define a series and test for convergence of a series in terms of the limit of the sequence of partial sums. This will include

a) geometric series with applications;
b) harmonic series;
c) alternating series with error bound;
d) terms of series as areas of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of p-series; and
e) ratio test for convergence and divergence. *

* For those students who are enrolled in AP Calculus BC.

APC.17 The student will define, restate, and apply Taylor series. This will include

a) Taylor polynomial approximations with graphical demonstration of convergence;
b) Maclaurin series and the general Taylor series centered at $x = a$;
c) Maclaurin series for the functions e^x, $\sin x$, $\cos x$, and $1/(1 - x)$;
d) formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series;
e) functions defined by power series;
f) radius and interval of convergence of power series; and
g) Lagrange error bound of a Taylor polynomial. *

* For those students who are enrolled in AP Calculus BC.